LASER INDUCED FLUORESCENCE/PARTICLE TRACKING SYSTEM FOR STRATIFIED FLOW RESEARCH

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by

H.J.S. Fernando
Department of Mechanical & Aerospace Engineering
Arizona State University
Tempe, AZ 85287-6106

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1. Introduction:

During the contract period the principal investigator and his associates, Dr. I.P.D. DeSilva and Mr. Jim McGrath, worked on several problems related to turbulence in stratified fluids. The results of these studies are outlined below. Relevant publications originated from the P.I.'s group during the contract period are listed at the end of this report.

The research conducted has two components. In the first part, a model that deals with turbulent mixing across sharp density interfaces subjected to homogeneous turbulence was developed. The analysis was essentially based on the rapid-distortion-theory approach of Hunt (1984, *J. Fluid Mech.*, 138, 161), which reckon on the empirical observation that the dissipation rate of turbulent kinetic energy (ε) above an interface is independent on the height z. The homogeneous turbulence was modeled using the classical von Karman spectra and the interfacial motions were assumed to be governed by the linear internal-wave equations. It was further assumed that the first mode of internal waves governs the interfacial motions, thus simplifying the interfacial boundary conditions; the latter have been a major difficulty in modeling fluid motions near sharp density interfaces.

Two configurations, namely, (*i*) a turbulent fluid layer separated from a non-turbulent heavy fluid layer by a density interface (single-sided stirring) and (*ii*) a density interface sandwiched between two layers of equal turbulent intensities and length scales (double-sided stirring), were considered. The former is applicable to situations of mixed-layer deepening in stably stratified natural water bodies due to convective stirring or surface-wave breaking whereas the latter is important in analyzing the interfacial migrations in density step structures -- for example, when multiple turbulent layers are separated by density interfaces, as in thermohaline staircases in the ocean (e.g., Kelley 1987, *J. Phys. Ocean.*, 17, 1633). Detailed measurements on the interfacial structure for the case (i) have been made by Hannoun & List (1988, *J. Fluid Mach.*, 189, 211), when the interface is forced by oscillating-grid induced shear free turbulence. Corresponding measurements for the case (ii) were undertaken in the second part of our study.

The existence of steady resonant modes at the interface was identified when $\omega < Ri/2$, where ω is the frequency of the waves, $Ri = \Delta b L_H / u_H^2$ is the interfacial Richardson number, Δb is the interfacial buoyancy jump and L_H is the integral lengthscale of turbulence. Unsteady analysis show that the growing waves efficiently absorb energy from the turbulent region at or near $\omega = Ri/2$, and the transfer of energy to other modes takes place thereafter. The resonant internal waves intermittently break down owing to local instabilities and dissipate energy thus establishing a quasi-stationary state at the interface with a finite internal-wave energy level. Such breaking events cannot be described by the linear theory and modeling based on physical and mathematical reasoning was introduced to close the problem. On the basis of the assumption (which is verified by the previous experimental observations) that the time-traces of the interfacial displacement at a given location has steep gradients, the following spectral form was proposed for the vertical velocity:

$$\psi_{33}(\omega,o) = \begin{cases} I(\omega) & \text{for } \omega > \omega_c \\ I(\omega_c) & \text{for } \omega < \omega_c \end{cases},$$

where $I(\omega)$ is the spectral form predicted by the linear theory and ω_c is the frequency below which the non-linear breaking of waves becomes important. The cut off frequency ω_c was evaluated using the physical argument that the ratio of the energy contained in the vertical velocity spectra in non-linear and linear regimes Γ is constant; this assumption led to the condition that $\mu_c = Ri/2\omega_c$ is a constant, independent of the Richardson number.

It was found that the experimental data on vertical velocity gives a good agreement with the predictions for case (i), if $\mu_c = 0.5$ (corresponding to $\Gamma = 1.25$) is chosen. The calculations for various quantities were carried out using $\mu_c = 0.5$ for both cases (i) and (ii). In calculating the displacement spectra using the above modeled spectra, the low frequency end near $\omega \to 0$ behaves as ω^{-2}, and thus the existence of a low frequency cut off, dictated by the lowest frequency of the motion in the mixed layer, was suggested. The proposed form of the displacement spectra was

$$\psi^d(\omega,0) = \begin{cases} \psi_{33}(\omega,0)/\omega^2 & \text{for } \omega > \omega_0 \\ \psi_{33}(\omega_0,0)/\omega_0^2 & \text{for } \omega < \omega_0 \end{cases}$$

Based on previous work, the non-dimensional ω_0 was selected as 0.18. Based on the above spectral forms, with $\mu_c = 0.5$, $\omega_0 = 0.18$ and the standard von Karman form of spectrum for homogeneous turbulence, the Richardson number Ri dependence of non-dimensional r.m.s. vertical velocity $\left(\overline{w^2}\right)^{1/2}$ and displacement $\left(\overline{\zeta^2}\right)^{1/2}$ as well as some other measurables were calculated; the energy absorbed to the interface was calculated using the unsteady linear theory. The principal predictions are as follows:

Single-Sided Stirring	Two-Sided Stirring
$\left(\overline{w^2}\right)^{1/2} = 1.83\ \mathrm{Ri}^{-1/3}$	$\left(\overline{w^2}\right)^{1/2} = 3.01\ \mathrm{Ri}^{-1/3}$
$\left(\overline{\zeta^2}\right)^{1/2} = 3.24\ \mathrm{Ri}^{-5/6}$	$\left(\overline{\zeta^2}\right)^{1/2} = 5.40\ \mathrm{Ri}^{-5/6}$
$\overline{\Delta pw} = 0.28\ \mathrm{Ri}^{-1/6}$	$\overline{\Delta pw(0)} = 0.14\ \mathrm{Ri}^{-1/6}$

However, it was pointed out that although the wave-number frequency spectrum $X_{ij}(k,\omega)$ of homogeneous turbulence used in the present work yields the correct form of Eulerian frequency spectra in the inertial subrange, the predicted magnitude is somewhat lower than previous experimental observations. Calculations performed with appropriate modifications to X_{ij} showed that the predictions based on $\Gamma = 1.0$ (corresponding to $\mu_c = 0.36$) also give good agreement with the experimental data. The resulting predictions are as follows (note that the unsteady-theory prediction for $\overline{\Delta pw}$ remains unchanged):

Single-Sided Stirring	Two-Sided Stirring
$\left(\overline{w^2}\right)^{1/2} = 1.80\ \mathrm{Ri}^{-0.33}$	$\left(\overline{w^2}\right)^{1/2} = 3.2\ \mathrm{Ri}^{-0.33}$
$\left(\overline{\xi^2}\right)^{1/2} = 3.24\ \mathrm{Ri}^{-0.83}$	$\left(\overline{\xi^2}\right)^{1/2} = 4.9\ \mathrm{Ri}^{-0.83}$

A phenomenological entrainment model was developed to calculate the entrainment rates across density interfaces. The model predictions were found to be in good agreement with the entrainment data obtained in previous oscillating-grid experiments of Nokes (1988, *J. Fluid Mech.*, 134, 195) and Turner (1968, *J. Fluid Mech.*, 33, 639). Turner (1968) conjectured that the entrainment rates for single and double-sided cases ought to be the same. One of the important findings of the present analysis is that these two cases are different and the different entrainment rates observed by Turner (1968) can be real; The differences arise from the fact that the profiles of buoyancy flux and the energy absorbed into the interface are dissimilar. The power-law dependencies of various quantities on the Richardson number Ri, however, were found to be similar.

The present analysis is complete than the previous theories on turbulent mixing across interfaces. Hannoun and List (1988) used the analysis of Phillips (1977, *Dynamics of the Upper Ocean, Cambridge Univ. Press*), based on the first mode of internal waves at sharp interfaces, to interpret their results but the latter does not take into account the effects of turbulence on the interface that lead to the resonant breaking of waves. Mory (1990, *J. Fluid Mech.*, 223, 193) assumed that K-H instability resulting from the sloshing motion of the eddies on the interface leads to mixing. The present analysis indicates that such a mechanism is unlikely to play an important role in mixing, except at small Ri.

In the second part of the investigation, a laboratory experimental study aimed at understanding turbulent mixing across shear-free density interfaces was conducted by employing state-of-the-art experimental techniques such as laser-induced fluorescence and two-point laser-Doppler velocimetry. The experimental configuration consisted of a sharp density interface sandwiched between either a turbulent layer and a non-turbulent layer (single-sided stirring) or two identical turbulent layers (double-sided stirring). The major focus was on the measurement of the magnitude of the r.m.s. interfacial velocity and r.m.s. interfacial displacement and their spectral forms as well as the fractal dimension, thickness and the local gradient Richardson number of the

interface. The mixing mechanisms operative at different Richardson number ranges were also studied by employing extensive quantitative flow visualization.

The results show that the mechanism of mixing differs for different Richardson number Ri ranges. When Ri < 15 for single-sided stirring or Ri < 20 for double-sided stirring the entrainment mechanism was found to be the impingement of eddies on the interface and splashing of the heavier fluid into the mixed layer. Wave activities were evident above these Ri values, but the waves were often found to be disturbed and pierced through by the eddies. At such intermediate Ri, the entrainment appears to be dominated by the shearing of interfacial-layer fluid by penetrating eddies with significant horizontal velocities. For Ri > 40 or so, the wave activities were prominent for both single and double-sided stirring cases and the breaking of these waves was the dominant entrainment mechanism.

In both cases the interfacial layer was found to be thin, with $h/L_H \approx 0.3$, and satisfies $Ri_I = \Delta bh/u_H^2 < \pi^2$, where h is the interfacial thickness. According to Carruthers and Hunt (1994, *J. Fluid Mech.*, Submitted), the latter implies that the interface should be dominated by the first mode of interfacial waves; velocity measurements corroborated this prediction. On the other hand when $Ri_I > \pi^2$ different modes of internal waves can be excited and a velocity maxima is possible within. This has been observed in the experiments of Perera et al. (1994, *J. Fluid Mech.*, In Press). Evidently, the difference in the internal-wave structure may cause different mixing rates across the interface.

The normalized r.m.s. turbulent velocity and interfacial-layer isopycnal displacements showed a variation with the bulk Richardson number as $(\overline{w^2})_i^{1/2}/u_H = 1.7\ Ri^{-1/3}$ and $(\overline{\zeta^2})_i^{1/2}/L_H = 3.2\ Ri^{-5/6}$, for the single-sided stirring and $(\overline{w^2})_i^{1/2}/u_H = 3.8\ Ri^{-1/3}$ and $(\overline{\zeta^2})_i^{1/2}/L_H = 4.5\ Ri^{-5/6}$, for the double-sided stirring. These results were found to be in good agreement with the predictions of the above model. Typically, the gradient Richardson number Ri_g within the interface was found to be much smaller than Ri, and fluctuates while frequently dropping below 1/4 and becoming negative at times. This implies the possibility of intermittent breaking of waves within the interface, which could be observed during the flow visualization studies. Apparently,

the dropping of Ri_g is concurrent with the increase of wave amplitude during resonant breaking of waves (see Phillips 1977).

Breaking waves produces a contorted (or fragmented) interface; the iso-pycnal surfaces within the interface may have approximate discontinuities, and hence the Fourier transformation of spatial and temporal interfacial displacement data is expected to produce k^{-2} (wave number) and ω^{-2} (frequency) spectra, respectively; the existence of such a spectral regime was verified experimentally. Note that the model described above employs this mathematical requirement for discontinuous functions to effect closure.

The fractal dimension D_k of selected isopycnal lines was determined as a measure of the fragmentation of the surface. Physically, highly contorted surfaces are expected to yield larger fractal dimensions and vice versa. The fractal dimensions of single and double-sided stirring were found to be the same for Ri < 40, in which parameter regime the interface is dominated by the inertial effects. When wave breaking takes over interfacial mixing at Ri > 40, the single and double-sided stirring showed different fractal dimensions for the same Ri. This shows that the interfaces for these two cases can have structural dissimilarities in the wave-breaking regime.

4. Publications During the Contract Period

Journal Papers

low Past a Sphere," *Physics of Fluids A*, **4(8)**, 1687-1696, 1992.

Noh, Y. and Fernando, H. J. S., "The Influence of Molecular Diffusion on the Deepening of the Mixed Layer," *Dynamics of Atmospheres and Oceans*, **17**, 187-215, 1993.

Noh, Y. and Fernando, H.J.S., "A Numerical Model for the Fluid Motion at a Density Front in the Presence of Background Turbulence," *Journal of Physical Oceanography.*, **23(6)**, 1142-1153, 1993.

Fernando, H.J.S. and De Silva, I.P.D., "Note on Secondary Flows in Oscillating-Grid Mixing Box Experiments," *Physics of Fluids A* , **5 (7)**, 1849-1851, 1993.

Fernando, H.J.S. and Ching, C.Y., "Effects of Background Rotation on Turbulent Line Plumes," *Journal of Physical Oceanography*, **23,** 2125-2129, 1993.

Fernando, H.J.S. and Ching, C.Y., "Lead-Induced Convection: A laboratory Perspective," *Journal of Marine Systems* , **4.**, 217-230, 1993.

Lin, Q., Boyer, D.L. and Fernando, H.J.S., "Note on Internal Waves Generated by the Turbulent Wake of a Sphere," *Experiments in Fluids* , **15**, 147-154, 1993.

Noh, Y. and Fernando, H.J.S., "The Transition in the Sedimentation Pattern of a Particle Cloud," *Physics of Fluids A* , 5 (12), 3049-3055, 1993.

Ching, C-Y., Fernando, H.J.S. and Noh, Y., Interaction of a Negatively Buoyant Line Plume with a Density Interface," *Dynamics of Atmospheres and Oceans*, (Accepted for Publication).

Davies, P.A., Boyer, D.L., Fernando, H.J.S. and Zhang, X., "On the Unsteady Motion of a Circular Cylinder Through a Linearly Stratified Fluid," *Philosophical Transactions of the Royal Society (London)* (Accepted for Publication).

Flor, J., Fernando, H.J.S. and Van Heijst, G.J.F., "The Evolution of an Isolated Turbulent Region in a Two-Layer Fluid," *Physics of Fluids* (Accepted for Publication).

Lin, Q., Boyer, D.L. and Fernando, H.J.S., "Flows Generated by the Periodic Horizontal Oscillations of a Sphere in a Linearly Stratified Fluid," *Journal of Fluid Mechanics* (Accepted for Publication).

Perera, H.J.S., Fernando, H.J.S. and Boyer, D.L., "Wave-turbulence Interaction at an Inversion Layer," *Journal of Fluid Mechanics*, (Accepted for Publication).

Lin, Q., Boyer, D.L. and Fernando, H.J.S., "The Vortex Shedding of a Streamwise-Oscillating Sphere Translating Through a Linearly Stratified Fluid," *Physics of Fluids* (Accepted for Publication)

DeSilva I.P.D. and Fernando, H.J.S., "Oscillating Grids as a Source of Nearly Isotropic Turbulence" *Physics of Fluids.* (Accepted for Publication)

Perera, M.J., Fernando, H.J.S. and Boyer, D.L., "Mixing Induced by the Oscillatory Flow Past a right Circular Cylinder," *Journal of Fluid Mechanics* (Accepted for Publication).

Papers Submitted

Fernando, H.J.S., van Heijst, G.J.F. and Fonseka, S.V., "The Evolution of an Isolated Turbulent Region in a Stratified Fluid," *Journal of Fluid Mechanics*, (under revision).

DeSilva I.P.D. and Fernando, H.J.S., "The Collapse of a Turbulent Mixed Region in a Stratified Fluid," *Journal of Fluid Mechanics* (under revision).

Ayotte, R.A. and Fernando, H.J.S., "The Motion of a Turbulent Thermal in the Presence of Background Rotation," *Journal of Atmospheric Sciences* (Under revision).

Davies, P.A., Mofor, L.A., and Fernando, H.J.S. "Laboratory Studies of Mixed Buoyant Jets in Shallow CrossFlows" *Transactions of the Institute of Civil Engineers*, U.K (Under revision)..

Fernando, H.J.S. "Migration of Density Interfaces Subjected to Differential Turbulent Forcing", Submitted to *Journal of Geophysical & Astrophysical Fluid Dynamics.*

Bermzn, N.S., Boyer, D.L., Brazel, A.J., Brazel, S.W., Celada, R.A., Chen, R-r, Fernando, H.J.S., Fitch, M.J., and Wang H-w. "Synoptic Classification and the Design of Physical Model Experiments for Complex Terrain," Submitted to *Journal of Applied Meteorology*

Papers in Preparation

Fernando, H.J.S. and Hunt, J.C.R. "Turbulent Mixing Across Shear Free Density Interfaces; Part 1 - Modeling Considerations," Submitted to *Journal of Fluid Mechanics*

Fernando, H.J.S., McGrath, J. and Hunt, J.C.R. "Turbulent Mixing Across Shear Free Density Interfaces; Part 2 - Laboratory Experiments," Submitted to *Journal of Fluid Mechanics*

Fernando, H.J.S., Mofor, L., Davies, P.A. and Ching, C.Y., "Interaction of Multiple Line Plumes in an Uniform Environment,"

Fernando, H.J.S., Ching, C.Y. and Stegen, G.R., "Some Aspects of the Evolution of Thermohaline Staircase Structures,"

Fernando, H.J.S., Ayotte, B.A. and Chen, R.-r., "Turbulent Plumes in Rotating Fluids"

DeSilva, I.P.D., Fernando, H.J.S., Montenegro, L. and Brandt, A.A. "Laboratory Experiments on Instabilities in Stratified Shear Flows"

Kit, E., Fernando, H.J.S., and Brown, G., "Measurement of Eulerian Frequency Spectrum in Homogeneous Turbulence"

Neves, J. and Fernando, H.J.S. "Behavior of a Sediment-Laden Vertical Turbulent Jet"

DeSilva, I.P.D., Fernando, H.J.S., McGrath, J. and Hebert, D. "Commotions in Oceans: A View From a Laboratory Platform"

Conference Proceedings

Fernando, H.J.S., Perera, H.J.S. and Boyer, D.L., "Trapping of Internal Gravity Waves in an Inversion", Proceedings, Eleventh Australasian Fluid Mechanics Conference, Hobart, 1165-1168, 1992.

Davies, P.A., Boyer, D.L., Fernando, H.J.S. and Zhang, X. On the Flow Generated by the Unsteady Motion of a Circular Cylinder Through a Linearly Stratified Fluid, Eleventh Australasian Fluid Mechanics Conference, Hobart, 1021-1024, 1992.

Fernando, H.J.S., Hunt, J.C.R. and Carruthers, D.J., Turbulence, Waves, and Mixing at Stratified Density Interfaces: Modelling and Experiments. Invited Paper, Fourth IMA Conference on Stably Stratified Flows (Ed. I. Castro and N. Rockliff), xxx-xxx, 1992.

Fernando, H.J.S., Perera, M.A.J.M. and McGrath J., "Turbulent Mixing Across Density Interfaces: Some New Concepts and New Results", XVIIIth International Congress of Theoretical and Applied Mechanics, Israel, 54-55, 1992.

Davies, P.A., Boyer, D.L., Fernando, H.J.S., and Zhang, X., "Wake Flows in Stratified Fluids," *Waves and Turbulence in Stratified Fluids*, (Ed. S.D. Mobbs and J.C.King), 301-322, Clarendon Press, 1993.

Noh, Y., and Fernando, H.J.S. "The Generation of Stratification in Shear Free Turbulence by an Imposed Buoyancy Flux," Near Wall Turbulent Flows (Ed. R.M.C. So, C.G.Speziale and B.E.Launder), Elsevier, 367-375, 1993.

Ayotte, B.A. and Fernando, H.J.S., "Laboratory Studies Related to Open-Ocean Deep Convection. Ninth Conference on Atmospheric and Oceanic Waves, San Antonio, Texas, 249-251, 1993.

Berman, N.S., Brazel, A.J., Boyer, D.L., Fernando, H.J.S., and Chen, R-r., "Air Quality in Nogales", Southwest Center for Environmental Research Policy Meeting, Mesa, AZ, March 26-27, 1993.

Berman, N.S., Chen R-r., Fernando, H.J.S., Boyer, D.L., and Celada, R.A., "Combined Physical and Numerical Modelling for the Analysis of Wind Fields in Complex Terrain", Proceedings of the Speciality Conference on "The role of Meteorology in Managing the Environment in the 90's, VIP-29, Air and Water Management Association, Pittsburg, PA, 296-303, 1993.

Fernando, H.J.S., Davies, P.A., Ayotte, B.A., Mofor, L.A., and Ching, C.Y., "Turbulent Plumes, Thermals and Convection in Oceans", NATO Advanced Research Workshop on "Recent Advances in the Fluid Mechanics of Turbulent Jets and Plumes", Vianna do Castello, Portugal, xxx-xxx, Kluwer Academic, 1993.

Etling, D. and Fernando, H.J.S., "On the Influence of Background Rotation on Turbulent Jets", NATO Advanced Research Workshop on "Recent Advances in the Fluid Mechanics of Turbulent Jets and Plumes", Vianna do Castello, Portugal, xxx-xxx, Kluwer Academic, 1993.

Conference Presentations/Abstracts

Hunt, J.C.R., Carruthers, D.J. and Fernando, H.J.S., Turbulence, Waves, and Mixing at Stratified Density Interfaces: Modelling and Experiments. Invited Paper, Fourth IMA Conference on Stably Stratified Flows - Flow and Dispersion Over Topography, University of Surry, Guilford, 21st-23rd Sept., 1-2, 1992, .

Fernando, H.J.S., Chen, R-r., and Ayotte, B. Laboratory Studies on Deep Ocean Convection. *Bull. Am. Phys. Soc.*, **37(8)**, 1806, 1992.

Lin, Q., Boyer, D.L., and Fernando, H.J.S. Flows Generated by the Periodic Oscillations of a Sphere in a Linearly Stratified Fluid. *Bull. Am. Phys. Soc.*, **37(8)**, 1746, 1992.

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Fernando, H.J.S., Stratified Turbulence: A Metaphor for Non-Equilibrium Complex Turbulent Flows. Invited Paper, ONR Workshop on Non-Equilibrium turbulence, Mar 10-12, Tempe, AZ, 88-89, 1993.

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Fernando, H.J.S. "Migration of Density Interfaces Subjected to Differential Turbulence Forcing", *Bull. Am. Phys. Soc.*, **38(12)**, 2254, 1993.

Ching, C.Y., and Fernando, H.J.S. "The Interaction of Turbulent Line Plumes" *Bull. Am. Phys. Soc.*, **38(12)**, 2253, 1993.

DeSilva, I.P.D., Montenegro, L.M., Mavrikos, N., Fernando, H.J.S. and Brandt, A. "Experiments on Kelvin-Helmholtz Instabilities in Stratified Shear Flows" *Bull. Am. Phys. Soc.*, **38(12)**, 2254, 1993.

Fernando, H.J.S., Chen R-r., Berman, N.S., Boyer,, D.L., and Brazel, T. "Change of Atmospheric general Circulation due to Global Warming: Inferences from a Physical Model," submitted to the *International Speciality Conference on Global Climate Change: Science, Policy and Mitigation Strategies* to be held in Phoenix, AZ, April 5-8, 1994.

Vorapayev, S.I., Fernando, H.J.S. and Mitchell, L.A., Pattern Interactions in Two-Dimensional Decaying Turbulence. Submitted to 5th European Turbulence Conference, 1994.